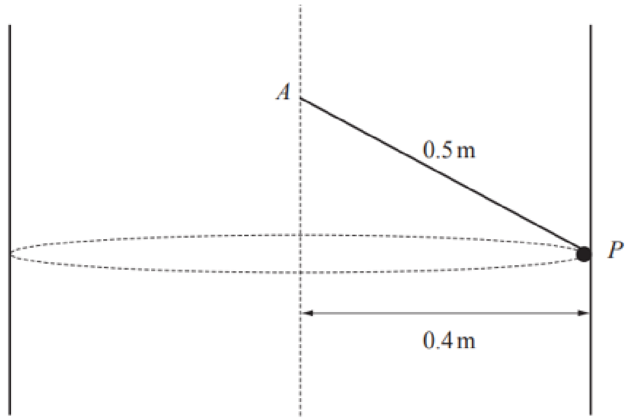
Problem Solving With Circular Motion

Q1, (OCR 4729, Jun 2013, Q5)

A vertical hollow cylinder of radius 0.4 m is rotating about its axis. A particle P is in contact with the rough inner surface of the cylinder. The cylinder and P rotate with the same constant angular speed. The coefficient of friction between P and the cylinder is μ.

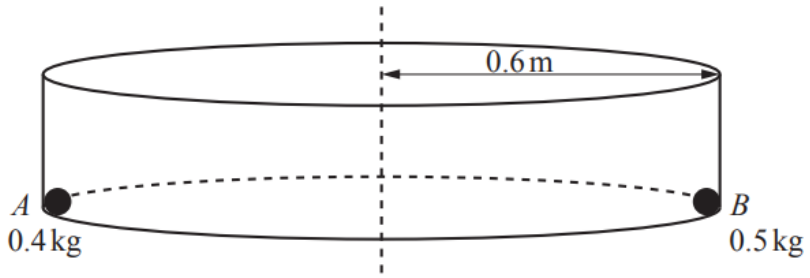
(i) Given that the angular speed of the cylinder is $7\,\text{rad s}^{-1}$ and P is on the point of moving downwards, find the value of μ. **[5]**

The particle is now attached to one end of a light inextensible string of length 0.5 m. The other end is fixed to a point A on the axis of the cylinder (see diagram).



(ii) Find the angular speed for which the contact force between P and the cylinder becomes zero. **[5]**

Q2, (OCR 4729, Jun 2015, Q8)



Two small spheres, A and B, are free to move on the inside of a smooth hollow cylinder, in such a way that they remain in contact with both the curved surface of the cylinder and its horizontal base. The mass of A is 0.4 kg, the mass of B is 0.5 kg and the radius of the cylinder is 0.6 m (see diagram). The coefficient of restitution between A and B is 0.35. Initially, A and B are at opposite ends of a diameter of the base of the cylinder with A travelling at a constant speed of $v\,\text{m s}^{-1}$ and B stationary. The magnitude of the force exerted on A by the curved surface of the cylinder is 6 N.

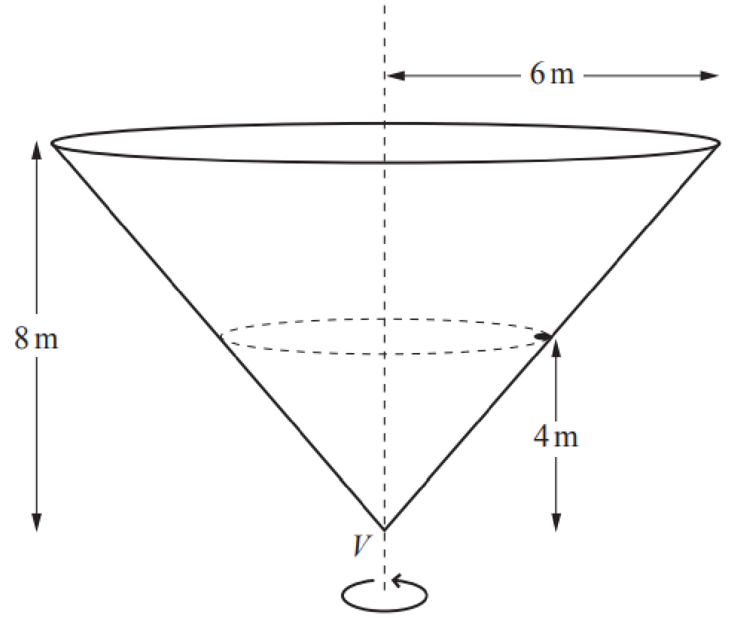
(i) Show that $v = 3$. **[2]**

(ii) Calculate the speeds of the particles after A's first impact with B. **[6]**

Sphere B is removed from the cylinder and sphere A is now set in motion with constant angular speed $\omega\,\text{rad s}^{-1}$. The magnitude of the total force exerted on A by the cylinder is 4.9 N.

(iii) Find ω. **[4]**

Q3, (OCR 4729, Jan 2013, Q8)



A conical shell has radius 6 m and height 8 m. The shell, with its vertex V downwards, is rotating about its vertical axis. A particle, of mass 0.4 kg, is in contact with the rough inner surface of the shell. The particle is 4 m above the level of V (see diagram). The particle and shell rotate with the same constant angular speed. The coefficient of friction between the particle and the shell is μ.

(i) The frictional force on the particle is F N, and the normal force of the shell on the particle is R N. It is given that the speed of the particle is 4.5 m s^{-1}, which is the smallest possible speed for the particle not to slip.

(a) By resolving vertically, show that $4F + 3R = 19.6$. **[2]**

(b) By finding another equation connecting F and R, find the values of F and R and show that $\mu = 0.336$, correct to 3 significant figures. **[6]**

(ii) Find the largest possible angular speed of the shell for which the particle does not slip. **[6]**

Q4, (Jun 2006, Q7)

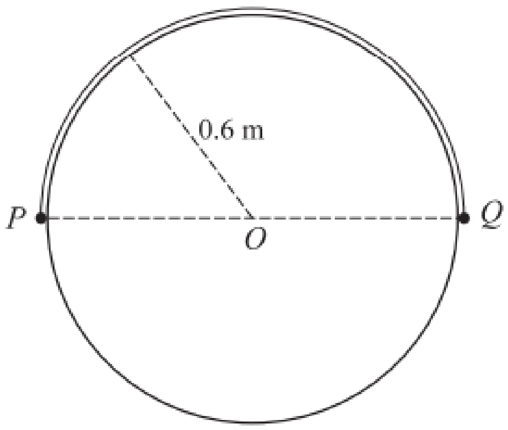


Fig. 1

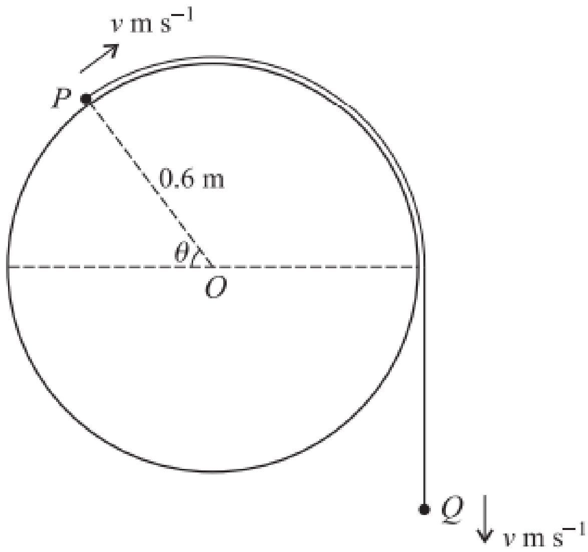


Fig. 2

A smooth horizontal cylinder of radius 0.6 m is fixed with its axis horizontal and passing through a fixed point O. A light inextensible string of length 0.6π m has particles P and Q, of masses 0.3 kg and 0.4 kg respectively, attached at its ends. The string passes over the cylinder and is held at rest with P, O and Q in a straight horizontal line (see Fig. 1). The string is released and Q begins to descend. When the line OP makes an angle θ radians, $0 \leqslant \theta \leqslant \frac{1}{2}\pi$, with the horizontal, the particles have speed $v\,\text{m s}^{-1}$ (see Fig. 2).

(i) By considering the total energy of the system, or otherwise, show that

$$v^2 = 6.72\theta - 5.04\sin\theta.$$ **[5]**

(ii) Show that the magnitude of the contact force between P and the cylinder is

$$(5.46\sin\theta - 3.36\theta) \text{ newtons.}$$

Hence find the value of θ for which the magnitude of the contact force is greatest. **[6]**

(iii) Find the transverse component of the acceleration of P in terms of θ. **[3]**

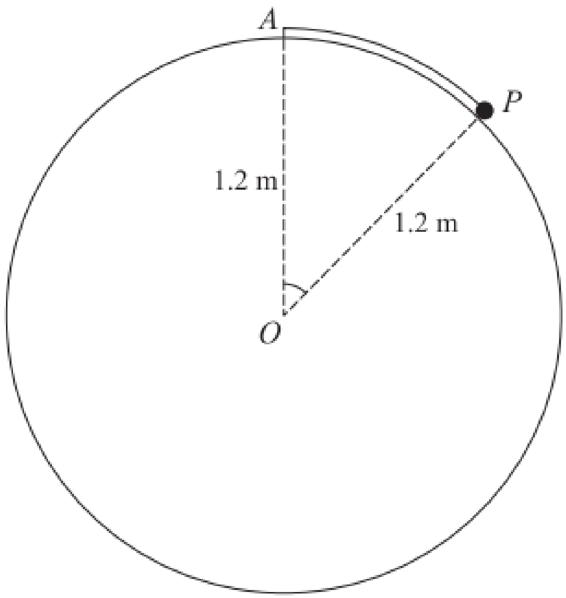
Q5, (Jun 2011, Q7i,ii)

One end of a light inextensible string of length 0.8 m is attached to a fixed point O. A particle P of mass 0.3 kg is attached to the other end of the string. P is projected horizontally from the point 0.8 m vertically below O with speed $5.6\,\text{m s}^{-1}$. P starts to move in a vertical circle with centre O. The speed of P is $v\,\text{m s}^{-1}$ when the string makes an angle θ with the downward vertical.

(i) While the string remains taut, show that $v^2 = 15.68(1 + \cos\theta)$, and find the tension in the string in terms of θ. **[7]**

(ii) For the instant when the string becomes slack, find the value of θ and the value of v. **[3]**

Q6, (Jan 2007, Q7)



The diagram shows a particle P of mass 0.5 kg attached to the highest point A of a fixed smooth sphere by a light elastic string. The sphere has centre O and radius 1.2 m. The string has natural length 0.6 m and modulus of elasticity 6.86 N. P is released from rest at a point on the surface of the sphere where the acute angle AOP is at least 0.5 radians.

(i) **(a)** For the case angle $AOP = \alpha$, P remains at rest. Show that $\sin\alpha = 2.8\alpha - 1.4$. [4]

(b) Use the iterative formula

$$\alpha_{n+1} = \frac{\sin\alpha_n}{2.8} + 0.5,$$

with $\alpha_1 = 0.8$, to find α correct to 2 significant figures. [2]

(ii) Given instead that angle $AOP = 0.5$ radians when P is released, find the speed of P when angle $AOP = 0.8$ radians, given that P is at all times in contact with the surface of the sphere. State whether the speed of P is increasing or decreasing when angle $AOP = 0.8$ radians. [7]