Probability Generating Functions

Q1, (2013, Q5)

The discrete random variable X has probability generating function given by

$$G_X(t) = k(5t^{-1} + 3 + 2t^2),$$

where k is a constant.

(i) Find

(a) the value of k, [1]

[1]

(ii) The random variables X_1 and X_2 are independent observations of X.

(a) Write down the probability generating function of Y, where $Y = X_1 + X_2$. [1]

(b) Use your answer to part **(ii)(a)** to find $E(Y)$ and $Var(Y)$. [8]

Q2, (2015, Q3)

The probability generating function of the random variable X is $\frac{1}{81}\left(t + \frac{2}{t}\right)^4$.

(i) Use the probability generating function to find $E(X)$ and $Var(X)$. [5]

(ii) The random variable Y is defined by $Y = \frac{1}{2}(X + 4)$. By finding the probability distribution of X, or otherwise, show that $Y \sim B(n, p)$, stating the values of n and p. [4]

Q3, (2016, Q3)

(i) Show that the probability generating function of a random variable with the distribution $B(n, p)$ is $(1 - p + pt)^n$. [3]

(ii) R and S are independent random variables with the distributions $B(8, \frac{1}{4})$ and $B(8, \frac{3}{4})$ respectively. Show that the probability generating function of $R + S$ can be expressed as

$$\left(\tfrac{3}{16} + \tfrac{1}{16}t(10 + 3t)\right)^8$$

and use this result to find $P(R + S = 1)$. [5]

Q4, (2017, Q2)

A discrete random variable X has the following probability distribution.

x	-1	2
$P(X = x)$	$\frac{1}{3}$	$\frac{2}{3}$

(i) Write down the probability generating function of X. [2]

(ii) T is the sum of ten independent observations of X. Use the probability generating function of T to find

(a) $E(T)$, [4]

(b) $P(T = 8)$. [3]

Q5, (2019 Specimen, Q1)

The discrete random variable X has probability generating function $G_X(t)$ given by

$$G_X(t) = at\left(t + \frac{1}{t}\right)^3,$$

where a is a constant.

(a) Find, in either order, the value of a and the set of values that X can take. [4]

(b) Find the value of $E(X)$. [2]